**The Garden Problem**

Objectives of this activity:

Students will understand patterns, write equations, find x-intercepts, the vertex is halfway between the x-intercepts, the vertex is the maximum value in this problem, how to find max values, and solve for any other given area.

Goal for this activity: Maximize the area of the garden.

Draw the picture:

House 25 feet of fencing is available to fence in the garden

X X

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Width (ft) | 1 | 3 | 5 | 7 | 9 | 11 |
| Length (ft) | 23 | 19 | 15 | 11 | 7 | 3 |
| Area (ft2) | 23 | 57 | 75 | 77 | 63 | 33 |

Questions to ask students and further their thinking:

What will happen to the length as the width increases?

What will happen to the area as the width increases and the length decreases?

As the width increases by 2 feet, the length will decrease by how much?

What happens numerically?

What will happen graphically?

What is the pattern between the width and the area?

3. Have students enter the data into their calculators. The width will go in L1 and the area will go in L2.

Clear out any equations in Y =.

Set the window based on the data in the table. No fat axes – reset the scale to fix the fat axis.

4. Define the variables: x = width in feet, y = the area in feet2

How do we represent the length in terms of x. What is the pattern in the table to help us here?

Length = 25 – 2x Width = x Area = x(25 – 2x) = 25x – 2x2  = -2x2 + 25x

Factored general or standard

5. Graph the function to make sure the function matches the data.

What are the x-intercepts using the calculator? x = 0, 12.5

How can we find the x-intercepts algebraically? This is where a discussion of the Zero Product Property will be very helpful.

We will set both factors from the factored form equal to zero to find x’s values on the x-axis; then solve for x.

6. The vertex is half-way between the two x-intercepts, so the halfway point would be 6.25. The width at the maximum area is 6.25; substitute this value into the function to get the area. (6.25, 78.125)

Go back to the calculator to find the maximum value. Press 2nd, Trace, maximum, place the cursor to the left of the maximum for the left bound, place the cursor to the right of the maximum for the right bound, press enter for Guess. (6.25, 78.125)

Finding the length for the maximum area: 25 – 2(6.25) = 12.5 feet

7. Finding the width and length when the area is 60 sq. feet.

Solve graphically: enter x(25 – 2x) in Y1 and 60 in Y2

Find the intersection points using 2nd, Trace, #5 x(width) = 3.24 and 9.26 ft.

Find the length: 25 – 2(3.24) = 18.52 ft. 25 – 2(9.26) = 6.48 ft.

Round to one decimal place: 18.5 ft. and 6.5 ft.

Solve algebraically: 0 = -2x2 + 25x – 60 Use the quadratic formula to solve for the width. Then find the length.